

# AN APPROACH TO ESTIMATING RIVER DISCHARGE FROM SPACE

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## Abstract

Multiple-regression analyses of hydraulic data from more than 1,000 discharge measurements, ranging in magnitude from over 200,000 to less than 1 m<sup>3</sup>/s, were used to develop generally applicable equations that use potentially observable variables to estimate river discharge using remote sensing techniques. Measurement uncertainty analysis indicates that existing satellite-based sensors can measure water-surface width (or surface area), water-surface elevation, and potentially the surface velocity of rivers with accuracies sufficient to provide estimates of discharge with average uncertainty on the order of 20 percent.

## 1. Measurement of Hydraulic Variables from Space and Estimation of River Discharge

Few studies have attempted to estimate river discharge from satellite information, although the potential

has been pointed out (Koblinsky, et al., 1993). Satellite-based sensors and other remote data sources can be used to determine water-surface width, water-surface elevation and slope, and channel morphology (University of Wisconsin, 2001). In addition, there is a possibility that surface velocity can be measured at discrete locations across the river channel (Vorosmarty et al., 1999; Emmitt, oral communication, 2001). Average depth and average cross-sectional velocity are the key hydrographic variables that cannot be directly measured from satellite information or other remote data sources are average depth and average cross-sectional velocity. Thus, average depth and average cross-sectional velocity will need to be related, at least implicitly, to stage and surface velocity, respectively, if these variables are used for estimating discharge.

There is a possibility that width, depth and velocity could all be measured or estimated simultaneously from data obtained from satellites. If so, discharge could be calculated directly from continuity as

$$Q = WYV, \quad (1)$$

where  $Q$  = discharge ( $L^3/T$ );

$W$  = width ( $L$ );

$Y$  = average depth ( $A/W$ );

$A$  = cross-sectional area ( $L^2$ ); and

$V$  = average velocity ( $L/T$ ).

The accuracy of the estimate is dependent on the accuracy and precision of the individual measurements of water-surface width, surface velocity, and stage, and on the estimations of mean velocity and mean depth derived from observations of surface velocity and stage. Because there is a potential that stage or surface velocity will not be observed with confidence (e.g. under strong winds or where topography obscures the signal), there will be many situations when all three of the key variables cannot be observed at the same time. In these situations, statistically based relationships may be useful.

## 2. Equations Used for Statistical Estimates of River Discharge

Predictable hydraulic geometry relationships between average velocity, average depth, width, and discharge in natural channels for within-bank flow conditions (Leopold et al., 1964) suggests the possibility that generally applicable discharge rating equations can be developed that use one or more of the elements of the hydraulic geometry as the predictor variable. Statistical studies by Riggs (1976), Jarrett (1984) and Dingman and Sharma (1997) have shown that reasonably accurate estimates of discharge for within-bank flows can be obtained without flow resistance as an input variable. Dingman and Sharma (1997) show that for a relatively wide range of river size and hydrologic conditions, discharge can be estimated as:

$$Q = 4.62A^{1.17} R^{0.40} S^{0.34}, \quad (2)$$

where  $Q$  = discharge ( $\text{m}^3/\text{s}$ );  
 $A$  = cross-sectional area ( $\text{m}^2$ );  
 $R$  = hydraulic radius (m) ( $A/P$ );  
 $P$  = wetted perimeter (m); and  
 $S$  = water surface slope (m/m).

Equation (2) was calibrated using over 500 flow measurements in 128 rivers with estimate accuracies, on average, of 20 percent or better. Equation (2) can be considered a generally applicable multivariate discharge rating. Assuming that the hydraulic radius is equivalent to the average depth,  $A^{1.17}$  can be substituted with  $W^{1.17}$ , and  $R^{0.40}$  can be substituted with  $Y^{1.57}$  with  $Y$  equal to the average depth (in meters). With this assumption, the form of equation (2) is hereafter referred to as Model 1. Because flow resistance is not an input variable, all of the necessary data can be measured either directly or remotely.

Alternatively, a relationship between discharge and an index velocity can be developed (Rantz, et al., 1982) that avoids the need to measure depth and slope but that still provides estimates over a wide range of flow conditions. The form of this rating equation would be

$$Q = cW^bV^f, \quad (3)$$

where  $V$  = the average cross-sectional velocity (m/s).

The coefficient  $c$ , and the exponents  $b$  and  $f$  reflect the correlation of depth with width and velocity. Assuming that the index velocity is the mean velocity, equation (3) is a simplified form of general continuity given by equation (1) that implies a predictable geomorphic relationship between width, depth and velocity, as suggested by Leopold et al. (1964). The form of equation (3) is hereafter referred to as Model 2. Hydraulic relations similar to Models 1 and 2 could be used to estimate discharge in rivers based on hydraulic information measured or inferred from space-based observations.

To explore the predictive characteristics of different combinations of potentially observable (or estimated) river hydraulic variables, a set of prediction equations (models) were developed based on Equations (1) and (2) from discharge measurement data using multiple regression analysis. The data base includes a total of 1,012 measurements in 102 rivers in North America and New Zealand. The measurement data consist of water-surface width or wetted perimeter, average water-surface depth or hydraulic radius, average velocity, and either water-surface slope measured concurrently with the discharge or slope obtained from topographic maps. Five hundred and sixty nine of the discharge measurements are reach-averaged (Barnes, 1967; Hicks and Mason, 1991; and Coon, 1998) that also include a measured water surface slope. The Hicks and Mason (1991) data includes the hydraulic radius and the wetted perimeter rather than the average depth and the water surface width (which could be obtained from the other data sources) which are reported and used from the other data sources (including Barnes, 1967, Coon, 1998). To provide consistency

between the data sets, the wetted perimeter is assumed equivalent to the water surface width, and the hydraulic radius is assumed equivalent to the hydraulic radius (Blodgett, 1986).

The remaining 443 measurements (representative of larger rivers), were obtained from the U.S. Geological Survey (USGS) NWIS data base (U.S. Geological Survey, 2001), and four measurements for the Amazon River at Obidos, Brazil from Oltman (1968) and Dury (1976). These large river discharge measurements report the average depth and water surface width, and are not reach-averaged; however it is anticipated that hydraulic variability between the measurement section and the reach as a whole is not large, and that the large number of observations will average out the variability. Additionally, the USGS discharge-measurement database does not include slope as a measured parameter; therefore, a channel slope for these river stations was measured from 1:24,000-scale USGS topographic maps. This results in a constant slope value for all of the flow measurements at a particular river station, implying slope as a geomorphic characteristic of the river. To provide data compatibility, the slope measurements associated with each discharge measurement from the reach averaged data were averaged over the range of flows at each river station, thus providing a constant characteristic slope for these rivers as well. The characteristic slope is thus transformed from a hydraulic variable to a discriminatory variable that represents a typical or average condition at each river station.

The implication of using a constant slope is explored by comparing two realizations of Model 1 developed from the reach-averaged data base, which includes a unique measured slope for all discharge measurements at each river station (Table 1). The first model realization assumes a variable slope and the second model realization assumes a constant slope. Comparison of the two regression models indicates nearly identical results (Table 1). Based on this comparison, we conclude that an average slope, or a channel slope that is constant for a river reach can be used in lieu of a measured slope, thus obviating the need to track water-surface slope as a dynamic prediction variable.

**Table 1** **COMPARISON OF REGRESSION MODELS USING CONSTANT AND VARIABLE SLOPE**

Model 1	Regression Statistics				Relative Residual	Log Residual	Actual Residual
	N	r <sup>2</sup>	RMSE		(Q* - Q)/Q	(logQ* - logQ)	(Q* - Q)
Realization 1 Variable Slope Q = 4.57*W <sup>1.18</sup> *Y <sup>1.74</sup> *S <sup>0.35</sup>	545	0.95	0.19	Mean	0.12	0.0009	14.6
Stdev				0.69	0.189	113.8	
Realization 2 Constant Slope Q = 4.60*W <sup>1.17</sup> *Y <sup>1.76</sup> *S <sup>0.35</sup>	545	0.95	0.2	Mean	0.14	-0.0012	11.4
Stdev				0.87	0.202	113	

N = number of observations  
r<sup>2</sup> = coefficient of determination  
RMSE = root mean square error of the estimates  
Q\* = predicted discharge

Based on these results, the entire database was used to calibrate and validate Models 1 and 2. The database was randomly divided into a calibration data set (N=506) and a validation data set (N=506). Using the calibration data set, the following regression models are developed:

Model 1:  $Q = 7.22(W)^{1.02}(Y)^{1.74}(S)^{0.35}$  (4)

Model 2:  $Q = 0.23(W)^{1.46}(V)^{1.39}$  (5)

The r<sup>2</sup> value (0.95) and RMSE (0.23) are the same for the two models. The validation data set was then used to test the predictive characteristics of the two models. The validation statistics are compared in Table 2. Figure 1 shows the predicted discharge (Q\*) plotted against the observed discharge (Q) for each model using the validation data set.

Model	Validation Statistics			
		Relative Residual (Q* - Q)/Q	Log Residual (logQ* - logQ)	Actual Residual (Q* - Q)
Model 1 Q = 7.22*W <sup>1.02</sup> *Y <sup>1.74</sup> *S <sup>0.35</sup>	Mean	0.16	0.004	243
	Stdev	0.81	0.207	5059
Model 2 Q = 0.23*W <sup>1.46</sup> *V <sup>1.39</sup>	Mean	0.1	-0.024	-790
	Stdev	0.71	0.231	9946

The log and actual residuals indicate that Model 1 tends to over-predict discharge and Model 2 tends to

under-predict discharge. The mean relative residual multiplied by 100 indicates the average percent error of the predictions. Model 2 performs the best in this regard, with an average relative error of 10 percent. Average relative error for Model 1 is less than 20 percent.

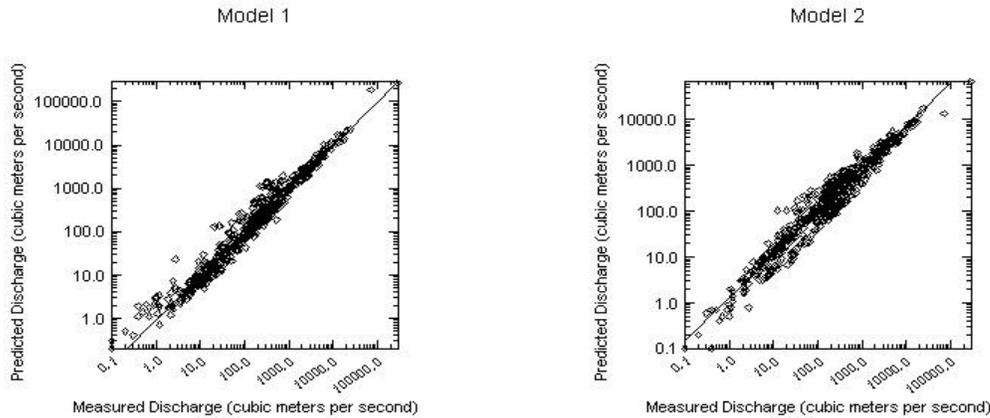


Figure 1 - Measured versus predicted discharge using Model 1 and 2 with the validation data set.

### 3. Measurement Uncertainty Analysis

Typical measurement accuracies were assigned to each variable, and then varied randomly assuming a normal distribution such that the mean measurement error for the entire database is zero and 95 percent of the errors are within the assigned accuracy. The modified data were used to re-estimate the discharge in the validation database using equation (1), Models 1 and 2. The resulting estimates were compared via the relative residual to the estimates that assumed no error. A maximum and minimum measurement uncertainty (error) is assumed for each dynamic variable. For  $W$ , the minimum assumed measurement uncertainty is 1 m, and the maximum uncertainty is 10 m, which would be consistent with many of the current synthetic-aperture radar (SAR) and visible spectrum sensors. The minimum assumed measurement uncertainty in water-surface elevation (as an index for  $Y$ ) is 0.1 m and the maximum is 0.5 m, consistent with the error range associated with current satellite altimeters (Birkett, 1998, Birkett et al., 2001). The minimum measurement uncertainty in  $V$  is assumed to be 0.1 m/s, which is the low end of the anticipated accuracy of a surface velocity measurement (Emmitt, personal communication, 2001), and the maximum assumed uncertainty was arbitrarily chosen to be 0.5 m/s (because the measurement of surface velocity from satellites has not been tested).

This analysis does not consider the potential uncertainty in estimating channel slope or the uncertainty associated with estimating the mean depth from stage, or the mean velocity from surface velocity. The

errors associated with estimating mean depth and velocity could be large, however these variables could be determined by a number of methods, including (1) estimation from topographic mapping and geomorphologic considerations, (2) measurement from repeated satellite observations over a range of flow conditions, and (3) measurement via field surveys.

The assumed measurement uncertainties are distributed with a mean of zero. For this reason, the standard deviation of the relative residuals is the best indicator of the impact of measurement uncertainty. The standard deviation of the relative residuals as a function of discharge category for the maximum assumed uncertainty, the minimum assumed uncertainty, and the case with no uncertainty are shown on Figure 2. The least variability is associated with using Equation (1) because there is no associated statistical uncertainty due to the fact that if  $W$  (water surface width),  $Y$  (average depth) and  $V$  (average velocity) are known continuity is preserved and there is no error. All of the plots in Figure 2 show that the impact of maximum measurement uncertainty on prediction variability, relative to the no uncertainty case, becomes pronounced below a discharge of  $10 \text{ m}^3/\text{s}$ . The impact of maximum uncertainty for discharge above  $10 \text{ m}^3/\text{s}$  is greatest for Equation (1) and Model 2. This result shows the effect of compounding uncertainty in the case of Equation (1), which includes uncertainty in all three dynamic variables, and indicates that uncertainty in  $V$  has a larger impact on prediction variability than does uncertainty in  $Y$  (comparing Model 1 and 2).

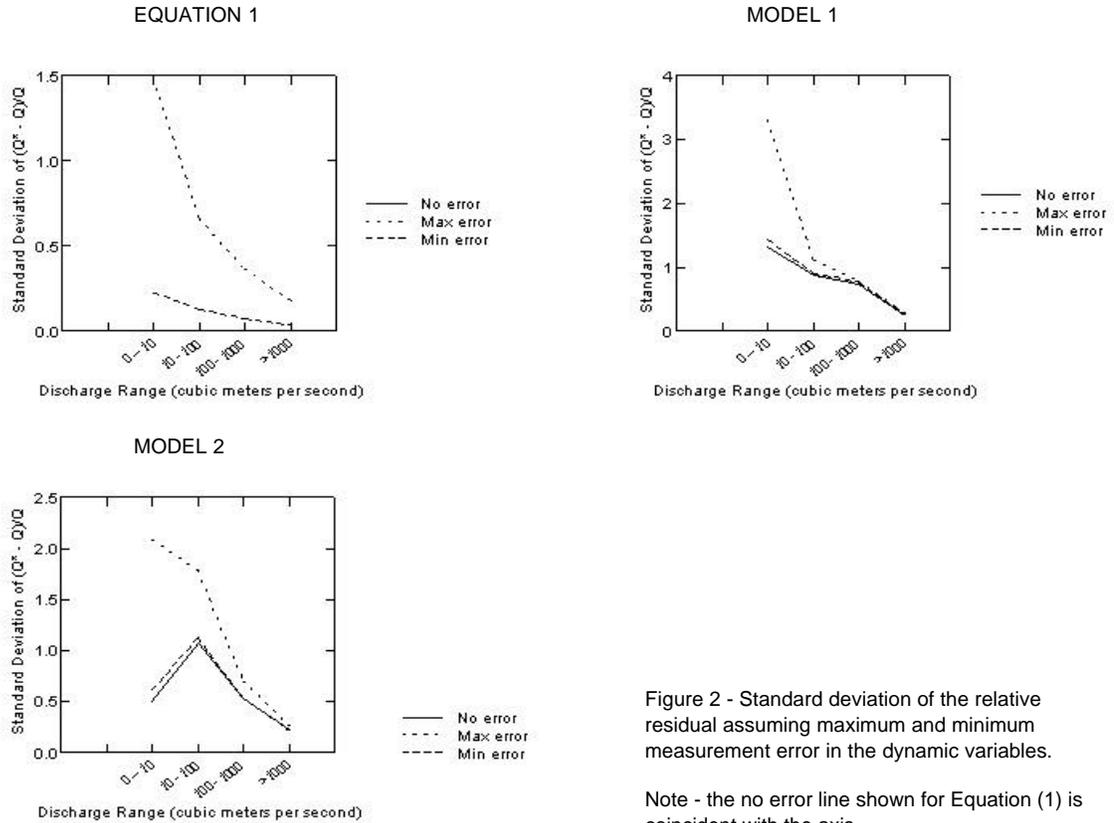


Figure 2 - Standard deviation of the relative residual assuming maximum and minimum measurement error in the dynamic variables.

Note - the no error line shown for Equation (1) is coincident with the axis.

The impact of minimum measurement uncertainty is not large within any discharge category, although as in the maximum uncertainty case it is most pronounced for discharge below 10 m<sup>3</sup>/s. However, if the minimum measurement uncertainty is achieved for all dynamic variables, predicting discharge with Equation (1) would result in a standard deviation of the relative residual (percent error) of less than 25 percent for discharge less than 10 m<sup>3</sup>/s, less than 15% for discharge in the range 10 - 100 m<sup>3</sup>/s and less than 10 percent for discharge greater than 100 m<sup>3</sup>/s. The impact of minimum measurement uncertainty using Models 1 and 2 is less than 15% for discharge less than 10 m<sup>3</sup>/s, and less than 10% for all other discharge categories. If the minimum measurement uncertainty can be achieved, uncertainty in the estimated discharge using the statistically based models is well below the uncertainty associated with the model itself (Figure 2).

## 6.0 References

- 1) Barnes, H. H., 1967. Roughness Characteristics of Natural Channels. USGS Water Supply Paper 1849. 213 p.
- 2) Birkett, C.M., 1998. Contribution of the TOPEX NASA radar altimeter to the global

- monitoring of large rivers and wetlands. *Water Resources Research* 34 (5): 1223-1239.
- 3) Birkett, C.M, L.A.K. Mertes, T. Dunne, M.H. Costa and M.J. Jasinski, 2001. Surface water dynamics in the Amazon Basin: application of satellite radar altimetry. in press.
  - 4) Blodgett, J.C, 1986. Rock riprap design for protection of stream channels near highway structures: Volume 1 – Hydraulic characteristics of open channels. U.S. Geological Survey Water Resources Investigation Report 86-4127, p. 20.
  - 5) Coon, W.F., 1998. Estimation of roughness coefficients for natural stream channels with vegetated banks. U.S. Geological Survey Water Supply Paper 2441. p. 133.
  - 6) Dingman, S.L. and K.P. Sharma, 1997. Statistical development and validation of discharge equations for natural channels. *Journal of Hydrology* 199:13-35.
  - 7) Dury, G.H., 1976. Discharge prediction, present and former, from channel dimensions. *Journal of Hydrology* 30 (3):219-245.
  - 8) Emmitt, G.D, 2001. Personal communication, Simpson Weather Associates, Charlottesville, VA..
  - 9) Hicks, D.M. and P.D. Mason, 1991. Roughness Characteristics of New Zealand Rivers. New Zealand DSIR Marine and Freshwater Resources Survey, Wellington, NZ. p. 329.
  - 10) Jarrett, R.D. (1984). Hydraulics of high gradient streams. *ASCE Journal of Hydraulic Engineering* 110:1519 - 1539.
  - 11) Koblinsky, C.J., R.T. Clarke, A.C. Brenner, and H. Frey, 1993. Measurement of river level with satellite altimetry. *Water Resources Research* 29(6):1839-1848.
  - 12) Leopold, L.B., M.G. Wolman, and J.P. Miller, 1964. Fluvial Processes in Geomorphology. W.H. Freeman, New York. p. 522.
  - 13) Oltman, R.E., 1968. Reconnaissance investigations of the discharge and water quality of the Amazon River. U.S. Geological Survey Circular 552. p. 16.
  - 14) Rantz, S.E., 1982. Measurement and computation of streamflow: Volume 1 measurement of stage and discharge. U.S. Geological Survey Water Supply Paper 2175. p. 284.
  - 15) Riggs, H. C., 1976. A simplified slope-area method for estimating flood discharges in natural channels. U.S. Geological Survey Journal of Research 4:285-291.
  - 16) University of Wisconsin Environmental Remote Sensing Center, 2001, <http://www.ersc.wisc.edu>.
  - 17) U.S. Geological Survey, 2001. Streamflow measurement data base, <http://water.usgs.gov/nwis/measurements>
  - 18) Vorosmarty, C., C. Birkett, L. Dingman, D.P. Lettenmaier, Yunjin Kim, Ernesto Rodriguez, George D. Emmitt, William Plant and Eric Wood, 1999. NASA post-2002 land surface hydrology mission component for surface water monitoring, HYDRA\_SAT. A report from the NASA Post 2002 LSHP Planning Workshop, Irvine CA April 12-14. p. 53.

